# UNRAVELLING ABSTRACT CYCLIC PROOFS INTO PROOFS BY INDUCTION

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Cyclic proof theory breaks tradition by allowing certain infinite proofs: those that satisfy a soundness condition and that can be represented by a finite cyclic graph. We reconcile cyclic proofs with traditional finite proofs: we extend abstract cyclic proof systems with a well-founded induction principle, and prune any cyclic proof into a finite proof in the extended system.

The abstract framework is designed to be general enough to cover dependent type theories, and is motivated by the hope to extend results relating proof assistants with more minimal versions of type theory [GMM06; CDP16]. In particular, the framework allows for cyclic proofs with a non-linear underlying inductive sort, and it covers recursive functions satisfying the size-change termination principle (used in proof assistant like `Agda`, `Dedukti`, and `Isabelle/HOL`). Such recursive functions can be viewed as cyclic proofs under the Curry-Howard correspondence:

| cyclic proof | recursive function |
|---|---|
| (co)inductive formula | (co)inductive type |
| cycle | recursive call |
| soundness condition | termination checking |

Our approach builds on Wehr [Weh25] and leverages an annotated representation of the cyclic proof, allowing us to extract induction hypotheses, and to determine their introduction order. We then replace each cycle with an appropriate application of the induction rule.

A preprint of the abstract framework and proof theoretic result is available [GO26]. We are currently working out the application to dependent type theory and hope to show some progress in the talk.

## References

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