Toward a Setoid Model of Oracle Computations*

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An oracle in type theory can be characterized by a type of queries $\mathcal{A} : \mathsf{Type}$ and, for each query $a : \mathcal{A}$, a type of answers $\mathcal{P}\ a : \mathsf{Type}$. Examples include classical principles such as the law of excluded middle, where

$$\mathcal{A} \equiv \mathsf{Prop} \qquad \text{and} \qquad \mathcal{P}\ \varphi \equiv \varphi \vee \neg\varphi$$

and the limited principle of omniscience, in which

$$\mathcal{A} \equiv \mathbb{N} \to \mathbb{2} \qquad \text{and} \qquad \mathcal{P}\ f \equiv \big(\exists(n : \mathbb{N}).\ f\ n = \mathtt{true}\big) \vee \forall(n : \mathbb{N}).\ f\ n = \mathtt{false}\ .$$

We consider a type theory where every (computation) type is oracle-computational, that is, for each type X, there is an operation

$$\mathrm{op}_X : \Pi_{(a:\mathcal{A})}.\ \big(\mathcal{P}\ a \to X\big) \to X\ .$$

Using this operation, we can construct a term in X by asking some queries from the oracle. We assume additional equations for these operations corresponding to the sheaf condition in [AB26].

In this talk, we present our recent progress on modeling this type theory of oracle computations using setoids in a simple CIC-style dependent type theory without quotient-inductive or higher-inductive types, cf. [AB26, Section 4] and [Swa24]. The main motivation is to implement such a type theory in popular frameworks such as Rocq or LEAN that do not support these features.

We focus on the case where the oracle is propositional in the sense that $\mathcal{P}\ a$ is subsingleton for each $a : \mathcal{A}$. We then discuss our approach towards generalizing this to general oracles where $\mathcal{P}\ a$ are proper types.

References

[AB26] Danel Ahman and Andrej Bauer. *Sheaves as oracle computations*. 2026. arXiv: 2602.22135 [math.LO].

[Swa24] Andrew W Swan. *Oracle modalities*. 2024. arXiv: 2406.05818 [math.LO].

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