ON THE SYNTAX OF CATEGORICAL STRUCTURES FOR DEPENDENT TYPES

HUGO HERBELIN

The proofs-as-programs correspondence explicitly connects specific logical systems to specific models of computation, e.g., Hilbert-style systems relate to combinatory logic, natural deduction to λ-calculus and sequent calculus to abstract machines. On the other side, when assigning an internal language to a categorical structure, the correspondence is in general not intended to be on the nose, because categorical structures are generally considered up to equivalence of presentation.

To get correspondences as fine-grained as in the case of the proofs-as-programs correspondence, we look at the question of assigning to categorical structures for dependent types the internal languages that reflect the exact choice of presentation made to define the structure. For instance, there are two standard ways to define comprehension in categorical structures for dependent types: using the mediating morphism of substitutions in a pullback as e.g. in the variants with chosen structure of finite-limit categories, display map categories, clans, categories with attributes, comprehension categories, or w-comonads, leading to the following kind of syntax for substitutions:

$$\sigma, \tau, \rho \quad ::= \quad ... \mid id \mid \tau \circ \sigma \mid p \mid \sigma^* \mid (\tau, \rho)_\sigma$$

or using dependent pairs of a substitution and a term, as in categories with families, natural models (up to Yoneda lemma), or generalised categories with families (when the underlying adjunction is characterised by the counit and the downwards direction of the hom-bijection), leading to the following kind of syntax for terms and substitutions:

$$t \quad ::= \quad ... \mid t[\sigma] \mid q \qquad\qquad \sigma, \tau, \rho \quad ::= \quad ... \mid id \mid \tau \circ \sigma \mid p \mid \langle \sigma, t \rangle$$

In the talk, we will attempt to make a systematic classification of the different degrees of freedom at hand when formulating the internal language canonically associated to the presentation of categorical models with chosen structure for dependent types (primitive dependent types and subtyping vs only selected slices and slice morphisms, explicit terms vs only sections, pseudofunctors vs cloven fibrations, split vs weak, pullbacks vs dependent pairs, Yoneda-like or morphism-like projections and variable, ...). In particular, regarding the choice between pullbacks and dependent pairs, we will rely on an analysis of the equivalence by Coraglia, Di Liberti and Emmeneger through the lense of an equivalence between comonads and adjunctions. Our approach is complementary to classifications up to equivalence (as, recently, in Ahrens, Lumsdaine and North), eventually providing a common setting for finely comparing type theories and their associated categorical structures.