

ON AUTO-SYNTHESIS

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In their celebrated paper *The Groupoid Interpretation of Type Theory* [HS95], Hofmann and Streicher demonstrate the independence of the *uniqueness of identity proofs* principle from the rules of Martin-Löf Type Theory by the construction of a countermodel, the groupoid model. The groupoid model accomplishes a peculiar meta-theoretic trick: it is a model “built out of groupoids”—the contexts are groupoids, the types are families of groupoids, etc.—but the type theory it models is a *synthetic theory of groupoids*—the identity types on a given type function like the hom-sets of a (1-)groupoid. Thus, in some sense, the groupoid model manifests the *auto-synthesis* of groupoids: a class of mathematical structures serving as the model of a synthetic theory of themselves.

Groupoids are not the only instance of this phenomena—the *setoid model* [Hof95; Alt99] gives a synthetic theory of setoids; the directed type theory of the *category model* [Nor19; NA25] gives a synthetic theory of categories [Neu25]—but examples are necessarily few. In this talk, we pose the general question: *what structures are capable of auto-synthesis?* We detail some structures which are apparently too simple—they lack the appropriate *fibrant, cosmological* character—as well as structures *more* complex than categories, and our attempts to make a study of their auto-synthetic potential tractable through computer mechanization.

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