TOPOLOGICAL PROPERTIES ARE LOGICAL PRINCIPLES IN TOPOLOGICAL MODELS

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In this talk I will relate topological properties of a T_0 space with validity of logical principles in the topos $\mathrm{Sh}(X)$ of sheaves over the space.

For example:

- A space is discrete iff the *law of the excluded middle* is valid in $\mathrm{Sh}(X)$.
- A second countable space is locally connected iff the *limited principle of omniscience* is valid in $\mathrm{Sh}(X)$.
- The lattice $\mathcal{C}(X, \mathbb{R})$ of continuous real-valued functions is conditionally complete iff the *weak law of the excluded middle* is valid in $\mathrm{Sh}(X)$.
- The *analytic Kripke schema*, which states that every truth value is equivalent to positivity of a Dedekind real number, holds over locally perfectly normal spaces.

I carry out calculations in Heyting-valued sets, as developed by Michael Fourman and Dana Scott [FS79], as these have simpler and more intuitive computation rules than the Kripke–Joyal semantics in sheaves. Thus I also review Heyting-valued sets from a categorical perspective and provide an internal language for them based on partial elements and the Heyting-valued singleton monad.

This talk is based on the author's Master's thesis [Str25], supervised by Andrej Bauer.

References

[FS79] Michael Paul Fourman and Dana Stewart Scott. "Sheaves and logic". In: *Applications of Sheaves*. Springer Berlin Heidelberg, 1979, pp. 302–401. ISBN: 9783540348498. DOI: 10.1007/bfb0061824.

[Str25] Luna Strah. "Topoloke lastnosti so logini principi v topolokih modelih: magistrsko delo". PhD thesis. 2025. DOI: 20.500.12556/RUL-173506.