FIRST ORDER LOGIC AS A SECOND ORDER GENERALISED ALGEBRAIC THEORY

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We give an algebraic description of first-order logic motivated by the CwF style presentation of type theory. We present first-order logic as a second order generalised algebraic theory (SOGAT) [Uem21]. This description features all the usual logical connectives and an intuitionistic natural deduction proof theory [HR04]. From the SOGAT description we derive a generalised algebraic theory (GAT) via the translation by Kaposi and Xie [KX24].

To show the notion of model we get is the expected one, we prove completeness for Beth models and define the double negation translation as a model construction from a classical version of our logic. We also show that the initial model (which can be called syntax) of first-order logic is a definable quotient. We formalised all the results in the Agda proof assistant.

References

[HR04] Michael Huth and Mark Dermot Ryan. *Logic in computer science - modelling and reasoning about systems (2. ed.)* Cambridge University Press, 2004.

[KX24] Ambrus Kaposi and Szumi Xie. "Second-Order Generalised Algebraic Theories: Signatures and First-Order Semantics". In: *9th International Conference on Formal Structures for Computation and Deduction (FSCD 2024)*. Ed. by Jakob Rehof. Vol. 299. Leibniz International Proceedings in Informatics (LIPIcs). Dagstuhl, Germany: Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2024, 10:1–10:24. ISBN: 978-3-95977-323-2. DOI: 10.4230/LIPIcs.FSCD.2024.10. URL: https://drops.dagstuhl.de/entities/document/10.4230/LIPIcs.FSCD.2024.10.

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