

DIMENSIONAL INFORMATION IN A TYPE THEORY WITH A DEFINITIONALLY PROOF IRRELEVANT LAYER

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We report on an implementation of a dependent type theory that supports definitional proof irrelevance. Our type theory is organized into two interwoven layers labelled by dimensions -1 and 0 , so that every type and every term is introduced at a specified dimension. This dimension determines the status of equality between terms, as follows. Types of dimension -1 are definitionally proof-irrelevant, hence we name them propositions. Types of dimension 0 have -1 -dimensional equality types and we name them sets. All our results are formalized in Agda [Agda] and our sources are available online¹.

Contexts may contain both propositions and sets, letting the two layers interact in a nontrivial way. We keep our type theory minimal, with only Pi-types, universes, and equality, and explore the behavior of type formers with respect the layers.

Our approach is close to that of relevance marks [Gil+19], where relevance information is local at binders. We take a geometric point of view and structure the syntax around the notion of dimension instead, following work of the second author [d’Esp23]. This allows us to state definitional proof irrelevance without referring to universes: any two -1 -dimensional terms of the same -1 -dimensional type are convertible.

Our implementation is derived from an implementation of a minimal type theory relying on an extrinsically typed, intrinsically scoped inductive-inductive syntax introduced by the first author [MW26]. With this approach, the dimensional information flows through the definitions almost always silently, except where layer-specific behavior is involved.

REFERENCES

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- [MW26] Julien Marquet-Wagner. “Induction-Induction for Intrinsically Well-Scoped Syntaxes”. In: *TYPES 2026* (2026). Forthcoming.

¹<https://github.com/thejohncrafter/SSTT26>